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Coulomb drag in the ballistic electron transport regime

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Abstract. We calculate the Coulomb drag current created in the ballistic transport regime in a one-dimensional nanowire by a ballistic current in a nearby nanowire. We predict sharp oscillations of the drag current as a function of the gate voltage or chemical potential. Our results may be of relevance to the issue of the cross-wire talk which is of pivotal importance to the proper operation of scaled-down devices and VLSI circuits.

The purpose of the present paper is to work out a theory of the Coulomb electron drag created in the course of ballistic electron transport in a nanowire due to a ballistic current in an adjacent nanowire. The possibility of observing the Coulomb drag effect was suggested by Pogrebinski [1]. Later on, the effect was considered for various geometries by a number of authors [2–7]. Of a vast number of papers in which the Coulomb drag has been treated, we refer here only to a few that are more closely related to the present one.

In the present paper we consider the Coulomb drag for the special case of so-called collisionless transport. The latter is treated in the spirit of the Landauer–Büttiker–Imry [8] approach to the electrical conduction in nanoscale structures. The electron motion through such nanostructures is collisionless, while nanostructures act effectively as waveguides. This is possible since the largest dimension of the structure is smaller than the electron mean free path in the problem (typically a few μm). Such nanoscale systems are characterized by low electron densities, which may be varied by means of the gate voltage. The transport of electrons in such a regime is called *ballistic* and is a quantum mechanical analogue of Sharvin's 3D classical point contact conductance [9].

The electron–phonon scattering in such wires leads to a number of effects that have been recently considered in the literature [10–14]. We will use a further development of the method worked out in these papers for consideration of the electron–electron interaction. This interaction within a single nanowire does not result in a current variation because of the quasimomentum conservation in the electron–electron collisions. However, if two such wires, 1 and 2, are near one another and are parallel, the Coulomb interaction of the electrons belonging to the different wires should result in the drag which we investigate in this paper. The drag force due to the ballistic current in wire 2 acts as a sort of permanent acceleration on the electrons of wire 1. As wire 1 has a finite length L, a steady drag current J is established in wire 1. In the present paper we calculate this current.

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Let us consider the conservation laws for the collisions of electrons belonging to two different wires, 1 and 2, each of them being parallel to the x-axis. We have

$$\epsilon^{(1)}_{np} + \epsilon^{(2)}_{n'p'} = \epsilon^{(1)}_{l,p+q} + \epsilon^{(2)}_{l',p'-q} \tag{1}$$

where $\epsilon^{(1,2)}_{np} = \epsilon^{(1,2)}_{n}(0) + p^2/2m$, m being the effective mass while n is the transverse quantization band (channel) index, with primed quantities corresponding to wire 2 throughout.

Introducing the notation

$$\delta\epsilon = \epsilon^{(1)}_{n}(0) + \epsilon^{(2)}_{n'}(0) - \epsilon^{(1)}_{l}(0) - \epsilon^{(2)}_{l'}(0) \tag{2}$$

the solution of equation (1) can be rewritten as

$$q = -(p-p')/2 \pm \sqrt{(p-p')^2/4 + m\,\delta\epsilon}. \tag{3}$$

We will assume that the electrons of the nanowires are degenerate and that the temperature is sufficiently low. The absolute values of the four quantities, namely, $\epsilon^{(1)}_{np}$, $\epsilon^{(2)}_{n'p'}$, $\epsilon^{(1)}_{l,p+q}$, and $\epsilon^{(2)}_{l',p'-q}$, should lie within the stripes $k_{\rm B}T$ around the corresponding Fermi levels. In other words, within the accuracy mk_BT/p_F, the following relations should be valid: $p = p^{(n)}_F$, $p' = p^{(n')}_F$, and

$$|p+q| = p^{(l)}_F \qquad |p'-q| = p^{(l')}_F. \tag{4}$$

Here $p^{(n)}_F$ denotes the Fermi quasimomentum for band n. In general it is impossible by variation of a single quantity, such as the transferred quasimomentum q, to satisfy both of the relations of equation (4) (provided, of course, that the distances between the channel bottoms are much bigger than $k_{\rm B}T$).

In other words, if the differences between the levels of transverse quantization are bigger than $k_{\rm B}T$ one cannot in general satisfy equation (1) for a finite $\delta\epsilon$. Therefore for a general case one should have $n = l, n' = l'$. If the two wires are identical, the pair of relations $n = l', n' = l$ is also possible. In both cases, $\delta\epsilon = 0$. We will assume the wires to be different. Then the δ-function

$$\delta(\epsilon^{(1)}_{np} + \epsilon^{(2)}_{n'p'} - \epsilon^{(1)}_{l,p+q} - \epsilon^{(2)}_{l',p'-q})$$

can be recast into the form

$$(m/|q|)\delta(p - p' + q). \tag{5}$$

This means that the quasimomentum transferred during a collision is $q = p' - p$. In other words, the electrons swap their quasimomenta as a result of collisions.

Assuming that the drag current in wire 1 is much smaller than the ballistic current in wire 2, we calculate it by solving the Boltzmann equation for wire 1 (otherwise we should have solved a system of coupled equations for the two wires). We have

$$v_{np}\frac{\partial F^{(1)}}{\partial p} = I^{(12)}\{F^{(1)}, F^{(2)}\} \tag{6}$$

where $F^{(1,2)}$ are the electron distribution functions in wires 1 and 2 respectively, and

$$\begin{aligned} I^{(12)}\{F^{(1)}, F^{(2)}\} = 2\sum_{p'}\sum_{q}\sum_{n'} & w(1, p+q, n; 2, p'-q, n' \leftarrow 1, p, n; 2, p', n') \\ & \times [F^{(1)}_{np}F^{(2)}_{n'p'}(1 - F^{(1)}_{n,p+q})(1 - F^{(2)}_{n'p'-q}) \\ & - F^{(1)}_{n,p+q}F^{(2)}_{n'p'-q}(1 - F^{(1)}_{np})(1 - F^{(2)}_{n'p'})]. \end{aligned} \tag{7}$$

Here, 2 is the spin factor; the scattering probabilities are assumed to be spin independent. If the electron–electron collisions can be described by perturbation theory, then the scattering probability is given by

$$w(1, p+q, n, 2, p'-q, n' \leftarrow 1, p, n; 2, p', n') = \frac{2\pi}{\hbar}|\langle 1, p+q, n; 2, p'-q, n'|V|1, p, n; 2, p', n'\rangle|^2 \times \delta(\epsilon^{(1)}_{np} + \epsilon^{(2)}_{n'p'} - \epsilon^{(1)}_{n,p+q} - \epsilon^{(2)}_{n',p'-q}). \quad (8)$$

The matrix element of the electron–electron interaction can be transformed into the form

$$\langle 1, p+q, n; 2, p'-q, n'|V|1, p, n; 2, p', n'\rangle = \frac{1}{L}\int \mathrm{d}^2 r_\perp \int \mathrm{d}^2 r'_\perp\ \phi^*_n(\boldsymbol{r}_\perp)\phi^*_{n'}(\boldsymbol{r}'_\perp)V_q(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp)\phi_n(\boldsymbol{r}_\perp)\phi_{n'}(\boldsymbol{r}'_\perp) \quad (9)$$

where

$$V_q = \int \mathrm{d}x\ V(x, \boldsymbol{r}_\perp)\exp(-\mathrm{i}qx/\hbar).$$

We have

$$\int \mathrm{d}x \int \mathrm{d}x'\ V(\boldsymbol{r} - \boldsymbol{r}')\exp[\mathrm{i}q(x - x')/\hbar] = 2e^2 L K_0(|q||\Delta r_\perp|/\hbar) \quad (10)$$

where $\Delta\boldsymbol{r}_\perp = \boldsymbol{r}_\perp - \boldsymbol{r}'_\perp$, and K_0 is a modified Bessel function defined in reference [15].

To calculate the current in wire 1, we iterate the Boltzmann equation for the electrons of the wire in the term describing the collisions between the electrons of wires 1 and 2. In this approximation one can choose the distribution functions in the collision term to be equilibrium ones. For the first wire we will denote them as $F^{(0)}_{np} = f(\epsilon^{(1)}_{np} - \mu)$, $F^{(0)}_{l,p+q} = f(\epsilon^{(1)}_{l,p+q} - \mu)$, and we will use analogous notation for the second wire.

We assume, in the spirit of the Landauer–Büttiker–Imry [8] approach, the quantum wire to be connected to reservoirs which we call 'left' (+) and 'right' (−), each of these being in independent equilibrium. Let the x-component of the quasimomentum of an electron in wire 2 before the scattering be p' and let that after the scattering by an electron of wire 1 be $p' - \hbar q$. Let $p' > 0$ while $p' - \hbar q < 0$. Then the first distribution function for wire 2 depends on the chemical potential $\mu^{(+)}$ while the second one depends on $\mu^{(-)}$, where $\mu^{(+)} - \mu^{(-)} = eV$. We assume that at $eV = 0$ the wires are in equilibrium. We denote the corresponding equilibrium chemical potential as μ.

Let us denote by $\Delta\{F\}$ the expression that one gets after substitution of the equilibrium distribution functions given above into the collision term. For $p' > 0$ ($p' < 0$) and $p' - q < 0$ ($p' - q > 0$), where p' is the electron quasimomentum in wire 2 before the scattering, we get

$$\Delta\{F^{(1)}, F^{(2)}\} = \pm 2\sinh\left(\frac{eV}{2k_B T}\right)\exp\left(\frac{\epsilon' - \mu}{k_B T}\right) \times [1 - f(\epsilon^{(1)}_{np} - \mu^{(+)})][1 - f(\epsilon^{(2)}_{n',p'} - \mu)]f(\epsilon^{(1)}_{n,p+q} - \mu^{(-)})f(\epsilon^{(2)}_{n',p'-q} - \mu) \quad (11)$$

where

$$\epsilon^{(1)}_{n,p+q} - \epsilon^{(1)}_{np} = \epsilon^{(2)}_{n',p'} - \epsilon^{(2)}_{n',p'-q} = (p'^2 - p^2)/2m \quad (12)$$

is the energy transferred in an electron–electron collision. The above expression is identically zero if the initial and final quasimomenta in wire 2 are of the same sign. At low

k_BT, it follows from the relation $q = p' - p$ that the initial and final quasimomenta, p and $p + q$ respectively, are of opposite sign as well.

As we are only interested in the Ohmic case, we will replace $\sinh(eV/2k_BT)$ by its argument and replace all of the chemical potentials in equation (11) by the same value μ. As a result, we get

$$\Delta\{F^{(1)}, F^{(2)}\} = \pm(eV/k_BT)[1 - f(\epsilon^{(1)}_{np} - \mu^{(+)})] \times [1 - f(\epsilon^{(2)}_{n',p'} - \mu)]f(\epsilon^{(1)}_{n,p+q} - \mu^{(-)})f(\epsilon^{(2)}_{n',p'-q} - \mu). \tag{13}$$

To calculate the current, we iterate the Boltzmann equation (6) in the collision term. The first iteration is

$$\Delta f = \left(\mp\frac{L}{2} + x\right)\frac{1}{v}I^{12}_{n,p} \tag{14}$$

for $p > 0$ and $p < 0$, respectively. The total drag current is

$$J = 2e\left(-\frac{L}{2} + x\right)\sum_n \int_0^\infty \frac{\mathrm{d}p}{2\pi\hbar}\ I^{12}_{n,p} + 2e\left(\frac{L}{2} + x\right)\sum_n \int_{-\infty}^0 \frac{\mathrm{d}p}{2\pi\hbar}\ I^{12}_{n,p} \tag{15}$$

where

$$I^{12}_{n,p} = \frac{16\pi e^4}{\hbar\kappa^2}\sum_{n'}\int\frac{\mathrm{d}p'}{2\pi\hbar}\int\frac{\mathrm{d}q}{2\pi\hbar}\ g_{nn'}(q)\Delta\{F^{(1)}, F^{(2)}\}\delta(\epsilon^{(1)}_{np} + \epsilon^{(2)}_{n'p'} - \epsilon^{(1)}_{l,p+\hbar q} - \epsilon^{(2)}_{l',p'-\hbar q}). \tag{16}$$

Here κ is the dielectric susceptibility of the lattice which we assume to be the same within and outside the nanowires, and

$$g_{nn'}(q) = \left|\int \mathrm{d}^2r_\perp \int \mathrm{d}^2r'_\perp\ |\phi_n(r_\perp)|^2|\phi_n(r'_\perp)|^2 K_0(q|\Delta\boldsymbol{r}_\perp|/\hbar)\right|^2. \tag{17}$$

The limits of the integration over q are determined by the requirement that p' and $p' - q$ be of different signs. As a result, we get the following equation for the drag current:

$$J = \sum_{nn'} J_{nn'}$$

where

$$J_{nn'} = \frac{4e^6V}{\pi^2\hbar^4k_BT}(j^{(1)}_{nn'} + j^{(2)}_{nn'} + j^{(3)}_{nn'} + j^{(4)}_{nn'}). \tag{18}$$

Here

$$j^{(1)}_{nn'} = \left(-\frac{L}{2} + x\right)\int_0^\infty \mathrm{d}p\ \int_0^\infty \mathrm{d}p' \int_{p'}^\infty \mathrm{d}q\ \ g_{nn'}(q)B(p, p'; p + q, p' - q)$$

$$j^{(2)}_{nn'} = -\left(-\frac{L}{2} + x\right)\int_0^\infty \mathrm{d}p \int_{-\infty}^0 \mathrm{d}p' \int_{-\infty}^{-p'} \mathrm{d}q\ \ g_{nn'}(q)B(p, p'; p + q, p' - q)$$

$$j^{(3)}_{nn'} = \left(\frac{L}{2} + x\right)\int_{-\infty}^0 \mathrm{d}p \int_0^\infty \mathrm{d}p' \int_{p'}^\infty \mathrm{d}q\ \ g_{nn'}(q)B(p, p'; p + q, p' - q)$$

$$j^{(4)}_{nn'} = -\left(\frac{L}{2} + x\right)\int_{-\infty}^0 \mathrm{d}p \int_{-\infty}^0 \mathrm{d}p' \int_{-\infty}^{-p'} \mathrm{d}q\ \ g_{nn'}(q)B(p, p'; p + q, p' - q)$$

and we have introduced the notation

$$B(p, p'; p + q, p' - q) = F_{np}(1 - F_{n,p+q})F_{n'p'}(1 - F_{n',p'-q})\delta(\epsilon^{(1)}_{np} + \epsilon^{(2)}_{n'p'} - \epsilon^{(1)}_{n,p+q} - \epsilon^{(2)}_{n',p'-q}). \tag{19}$$

After a transformation of the integration variables, the terms proportional to x cancel (as they should because of charge conservation), while the terms proportional to L add in the following way:

$$j^{(1)}_{nn'} + j^{(2)}_{nn'} + j^{(3)}_{nn'} + j^{(4)}_{nn'} = L \int_0^\infty \mathrm{d}p \left[\int_{-\infty}^0 \mathrm{d}p' \int_{-\infty}^{-p'} \mathrm{d}q\ g_{nn'}(q) B(p, p'; p+q, p'-q) - \int_0^\infty \mathrm{d}p' \int_{p'}^\infty \mathrm{d}q\ g_{nn'}(q) B(p, p'; p+q, p'-q) \right]. \qquad (20)$$

This expression can be calculated using the transformation of the δ-function given by equation (5). In this way one can see that the second term in the square brackets in equation (20) vanishes while the first term, after a transformation, gives for the drag current

$$J = \frac{4e^6 m V L}{\pi^2 \kappa^2 \hbar^4 k_\mathrm{B} T} \sum_{nn'} \int_0^\infty \mathrm{d}p\ q \int_0^\infty \mathrm{d}p'\ \frac{1}{p+p'} g_{nn'}(p+p') \times f(\epsilon^{(1)}_{np} - \mu)[1 - f(\epsilon^{(1)}_{np'} - \mu)] f(\epsilon^{(2)}_{n'p'} - \mu)[1 - f(\epsilon^{(2)}_{n'p} - \mu)]. \qquad (21)$$

Let us at first consider the terms of the sum in equation (21) with $\epsilon^{(1)}_n(0) \neq \epsilon^{(2)}(0)$. We number the levels of transverse quantization up from the lowest one. For definiteness, let us assume that $\epsilon^{(1)}_n(0) > \epsilon^{(2)}_{n'}(0)$. We will also assume that the differences $\epsilon^{(1)}_n(0) - \epsilon^{(2)}_{n'}(0)$ are much bigger than $k_\mathrm{B}T$. Then $f(\epsilon^{(1)}_{np} - \mu)$ can be considered as a step function, and the limits of the integral over dp are determined by the product $f(\epsilon^{(1)}_{np} - \mu)[1 - f(\epsilon^{(2)}_{n',p} - \mu)]$. The first factor determines the upper limit as

$$p^{(n)}_F = \sqrt{2m[\mu - \epsilon^{(1)}_n(0)]}.$$

The second factor gives the lower limit as

$$p^{(n')}_F = \sqrt{2m[\mu - \epsilon^{(2)}_{n'}(0)]}.$$

At the same time one can see that the factor $f(\epsilon^{(2)}_{n'p'} - \mu)[1 - f(\epsilon^{(1)}_{n,p'} - \mu)]$ and therefore also the integral over dp' vanish within the accepted approximation. For the same reason, the integral over dp vanishes for $\epsilon^{(2)}_{n'}(0) > \epsilon^{(1)}_n(0)$.

Now we are left with the terms of the sum with $\epsilon^{(1)}_n(0) = \epsilon^{(2)}_{n'}(0)$ (the equality should be fulfilled at least with an accuracy of the order of $k_\mathrm{B}T$—see below). To calculate these terms one cannot use the step function approximation, as the integral in equation (21) is dominated by the contribution of thermal layer near the Fermi level. To calculate the integrals over dp and dp', we will use the following identity:

$$f(\epsilon_{np} - \mu)[1 - f(\epsilon_{n,p} - \mu)] = -k_B T \frac{\partial f(\epsilon_{np} - \mu)}{\partial \epsilon_{np}}. \qquad (22)$$

This is a sharply peaked function of the electron energy. Provided that the rest of the functions in the integrand are smooth on the energy scale $k_\mathrm{B}T$, they can be taken out of the integrand at $p = p_F$. As a result, for every coincidence

$$\epsilon^{(1)}_n(0) = \epsilon^{(2)}_{n'}(0) \qquad (23)$$

we get the following contribution to the Coulomb drag current:

$$J = \sum_{nn'} J^{(0)}_{nn'} \qquad (24)$$

where

$$J_{nn'}^{(0)} = \frac{e^5 m^3 L k_{\mathrm{B}} T e V}{2\pi^2 \kappa^2 \hbar^4} \frac{1}{p_n^3} g_{nn'}(2p_n) \tag{25}$$

in which the channel quasimomentum is

$$p_n = \sqrt{2m[\mu - \epsilon_n^{(1)}(0)]}.$$

For the case in which there is no full coincidence (i.e. $\epsilon_n^{(1)}(0) \neq \epsilon_{n'}^{(2)}(0)$), we can generalize our calculation to include temperature dependence of the drag current. Assuming that there is only one pair of coinciding levels, and therefore omitting the summation over n, and taking out of the integral all of the slowly varying functions, we get

$$J = J^{(0)} \mathcal{L}(a, b) \mathcal{L}(b, a)$$

where $a = \epsilon_n^{(1)}(0)/k_{\mathrm{B}}T$ while $b = \epsilon_{n'}^{(2)}(0)/k_{\mathrm{B}}T$. Here

$$\mathcal{L}(a, b) = \int_{-\infty}^{\infty} \frac{\mathrm{e}^{x-b}\, \mathrm{d}x}{(1 + \mathrm{e}^{x-a})\mathrm{e}^{x-b}} = \frac{a - b}{1 - \mathrm{e}^{b-a}}.$$

Finally, we get for the drag current

$$J = \frac{e^5 m^3 L k_{\mathrm{B}} T e V}{2\pi^2 \kappa^2 \hbar^4} \frac{1}{{p_n}^3} g_{nn}(2p_n) \frac{[\epsilon_n^{(1)}(0) - \epsilon_{n'}^{(2)}(0)]^2}{4(k_{\mathrm{B}}T)^2} \left[\sinh \frac{\epsilon_n^{(1)}(0) - \epsilon_{n'}^{(2)}(0)}{2k_{\mathrm{B}}T} \right]^{-2}. \tag{26}$$

The ratio of the drag current to the ballistic current

$$J_b = \mathcal{N} \frac{e^2}{\pi\hbar} V \tag{27}$$

where $\mathcal{N}$ is the number of active channels (i.e. of 1D bands whose bottoms are below the Fermi level) is given for $\epsilon_n^{(1)}(0) = \epsilon_{n'}^{(2)}(0)$ by

$$\frac{J}{J_b} = \frac{e^4 m^3 L k_{\mathrm{B}} T}{2\pi\hbar^3 \kappa^2 \mathcal{N}} \sum_{nn'} \frac{1}{p_n^3} g_{nn'}(2p_n). \tag{28}$$

Equations (25)–(28) were also obtained by us using the quantum linear response theory of reference [13]. Including elastic scattering leads to the replacement of $g_{nn'}$ by $g_{nn'} T_n^{(1)} T_{n'}^{(2)}$ where $T_n^{(i)}$ is the transmission probability. We also showed that the reflections do not change the nullity of the effect on current of the Coulomb interactions within one nanowire.

There are no experiments yet on wire-to-wire coulomb drag, to our knowledge. Two recent experimental papers, [16] and [17], on the plane-to-plane Coulomb drag also include a number of references on that.

The subband structure is controlled by the gate voltage which changes the effective wire channel width and hence a threshold of propagation in each subband. In general the variation of the gate voltage may affect the widths and therefore the positions of the levels of transverse quantization in the two wires in different ways. As a result, in the course of the gate voltage variation a coincidence of a pair of such levels in the two wires may be reached. The estimate (28) is not very sensitive to the forms of the confining potential and electron densities. For the estimate, we assume the potential to be rectangular and $\mu = 14$ meV, $T = 1$ K, $W_2 = 42$ nm, $L = 1$ μm, $\kappa = 13$. The spacing between the wires is assumed to be 50 nm. In figure 1 the ratio in (28) is plotted as a function of the ratio of the effective wire widths. This plot exhibits striking oscillations with gigantic peak-to-valley ratios. The peaks occur when the channel velocities in two interacting wires are equal, which happens whenever any two current-carrying channels line up. This sort of

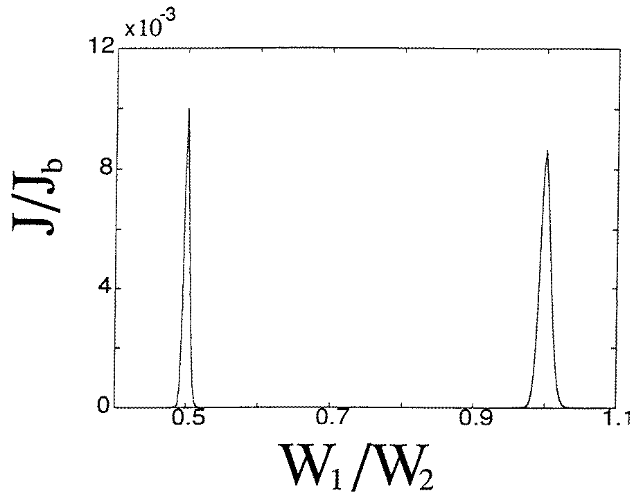

Figure 1. J/J_b is plotted as a function of W_1/W_2 where the width of wire 1 is controlled through the gate voltage ($\mu = 14$ meV, $T = 1$ K, $W_2 = 42$ nm, $L = 1$ μm, $\kappa = 13$, and the spacing between the wires is 50 nm).

coupling is particularly strong when such channel velocities are also quite small. The width of wire 1 can be controlled through the gate voltage.

To obtain an order-of-magnitude estimate, we can consider the following example. A current of 10 μA in wire 1 is realistic, and under favourable conditions can produce a current as large as 10^{-1} μA in wire 2 at special gate voltages (see figure 1). The latter value can be easily measured in experiment.

In this paper we have investigated a Coulomb drag mutually affecting quantum wires. We predict giant oscillations in the drag current caused in one wire by an electrical current in the other, as a function of the gate voltage. The peaks of the giant oscillations in the drag current occur whenever the channel velocities of the electrons near the Fermi levels in the two different wires coincide. (The importance of equal channel velocities was also pointed out by Vasilopoulos and Sirenko in reference [7], where they calculated a frictional force caused by Coulomb drag.) These peaks are particularly strong if the coinciding channel velocities are rather small. At the same time, they are assumed to be sufficiently big that for our approach the perturbation theory is still applicable. The coincidence of the channel velocities in the two wires can be achieved by variation of the gate voltage.

In the case considered, wire 2 is a part of a standard structure for measurement of a ballistic conductance, i.e. it joins two classical reservoirs, each of them being in independent equilibrium. However, wire 1 can be, for example, a part of a ballistic short-circuited structure, in which the value of the drag current may be found by measuring a corresponding magnetic flux.

Our results have several consequences: firstly, in connection with the issue of cross-wire talk which is of pivotal importance to the proper operation of scaled-down devices and VLSI circuits, according to our findings, one should be able to engineer circuitry so as to minimize the effects of cross-wire talk (for instance, by avoiding the level coincidence (26) as much as possible); secondly, one can investigate band structure, making use of the extreme sensitivity of the lining up of the levels; and lastly, this effect may play an important role in the direct investigation of Coulomb scattering.

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